

Static Charged Fluid in $(2+1)$ -Dimensions Admitting Conformal Killing Vectors

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New solutions for $(2+1)$ -dimensional Einstein-Maxwell space-time are found for a static spherically symmetric charged fluid distribution with the additional condition of allowing conformal killing vectors (CKV). We discuss physical properties of the fluid parameters. Moreover, it is shown that the model actually represents two structures, namely (i) *Gravastar* as an alternative of black hole and (ii) *Electromagnetic Mass* model depending on the nature of the equation state of the fluid. Here the gravitational mass originates from electromagnetic field alone. The solutions are matched with the exterior region of the Bañados-Teitelboim-Zanelli (BTZ) type isotropic static charged black hole as a consequence of junction conditions. We have shown that the central charge density is dependent on the value of M_0 , the conserved mass of the BTZ black hole. This in turn depends on the black hole event horizon which again is related to the Hawking radiation temperature of a BTZ black hole. Thus one may have a clue that the central charge density is related with Hawking radiation temperature of the BTZ black hole on the exterior region of the static charged fluid sphere.

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I. INTRODUCTION

The solutions of the charged fluid distribution in $(2+1)$ -dimensional gravity has become a subject of considerable interest. We search for some new solutions admitting conformal motion of Killing Vectors (CKV). However, to explore a natural relationship between space-time geometry and matter distribution for a star it is an usual practice to take into account the well-known inheritance symmetry. In this paper, we therefore investigate the solution of Einstein-Maxwell field equations in $(2+1)$ -dimension by using CKV. Matching conditions of the spherical charged fluid are imposed on charged BTZ type black hole in $(2+1)$ -dimensional space-time [1–3].

In this connection we note that study of *Gravastar* i.e. *Gravitational vacuum star*, have recently received tremendous impetus. It was proposed by Mazur and Mottola [4, 5] as an alternative to black holes by constructing a new type of solution for the endpoint of a gravitationally collapsing compact star. In the physical standpoint they extended the concept of Bose-

Einstein condensate to gravitational systems. The original Mazur-Mottola model [4, 5] contains an isotropic de Sitter vacuum in the interior whereas the exterior is a Schwarzschild geometry. The system was separated by a thin shell of stiff matter such that the configuration of a gravastar has three different regions: (I) Interior: $0 \leq r < r_1$, $p = -\rho$; (II) Shell: $r_1 < r < r_2$, $p = +\rho$; and (III) Exterior: $r_2 < r$, $p = \rho = 0$. It was argued by Mazur and Mottola [4, 5] that the presence of matter on the thin shell is required to achieve the required stability of systems under expansion by exerting an inward force to balance the repulsion from within. However, Usmani et al. [6] proposed a charged gravastar admitting conformal motion which facilitates stability of a fluid sphere by avoiding gravitational collapse [7–11]. Some other kind of Gravastar solutions are available in the literature which demand special mention in connection to our present investigation [12–16].

The scheme of the investigation is as follows. In the next Sec. II, we provide Einstein-Maxwell field equations: firstly, in the compactified implicit forms and secondly, in the expanded explicit forms for isotropic, static, spherically symmetric charged fluid in $(2+1)$ -dimensional space-time. We explain the interior region of the sphere under CKV which represents two different structures of the model namely (i) Gravastar and (ii) Electromagnetic Mass model. We rewrite the fluid parameters (ρ, p_r) and metric coefficients (e^γ, e^λ) in the appropriate forms to attain physical structures in the Sec. III whereas the

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Sec. IV gives the exterior region and Sec. V matches the interior to the BTZ-type exterior of the sphere of radius R . The Sec. VI is used as the platform for providing some concluding remarks.

II. EINSTEIN-MAXWELL FIELD EQUATIONS

The line element for the interior space-time of a static spherically symmetric charged distribution of matter in $(2+1)$ -dimensions is taken as [2, 3]

$$ds^2 = -e^{\gamma(r)} dt^2 + e^{\lambda(r)} dr^2 + r^2 d\theta^2. \quad (1)$$

To get Einstein-Maxwell equations, we write the Einstein-Hilbert action coupled to electromagnetism in the following form

$$I = \int dx^3 \sqrt{-g} \left(\frac{R - 2\Lambda}{16\pi} - \frac{1}{4} F_a^c F_{bc} + L_m \right), \quad (2)$$

where L_m is the Lagrangian for matter. The variation with respect to the fundamental tensor yields the following self consistent Einstein-Maxwell equations with cosmological constant Λ for a charged perfect fluid distribution:

$$R_{ab} - \frac{1}{2} R g_{ab} + \Lambda g_{ab} = -8\pi (T_{ab}^{PF} + T_{ab}^{EM}). \quad (3)$$

The explicit forms of the energy momentum tensor components for the perfect fluid and electromagnetic fields are given by:

$$T_{ab}^{PF} = (\rho + p) u_i u_k + p g_{ik}, \quad (4)$$

$$T_{ab}^{EM} = -\frac{1}{4\pi} \left(F_a^c F_{bc} - \frac{1}{4} g_{ab} F_{cd} F^{cd} \right), \quad (5)$$

where ρ , p , u_i are, respectively, matter-energy density, fluid pressure and three velocity vector of a fluid element. Here, electromagnetic field F_{ab} is related to current three vector as

$$F_{;b}^{ab} = -4\pi J^a, \quad (6)$$

with

$$J^c = \sigma(r) u^c, \quad (7)$$

where $\sigma(r)$ is the proper charge density of the distribution.

For this study we assume three velocity as $u_a = \delta_a^t$ and concerning the electromagnetic field tensor is given by

$$F_{ab} = E(r) (\delta_a^t \delta_b^r - \delta_a^r \delta_b^t), \quad (8)$$

where $E(r)$ is the electric field.

Hence the Einstein-Maxwell field equations with cosmological constant ($\Lambda < 0$) under the space-time metric (1) can be explicitly provided as (rendering $G = c = 1$)

$$\frac{\lambda' e^{-\lambda}}{2r} = 8\pi\rho + E^2 + \Lambda, \quad (9)$$

$$\frac{\gamma' e^{-\lambda}}{2r} = 8\pi p - E^2 - \Lambda, \quad (10)$$

$$\frac{e^{-\lambda}}{2} \left(\frac{1}{2} \gamma'^2 + \gamma'' - \frac{1}{2} \gamma' \lambda' \right) = 8\pi p + E^2 - \Lambda, \quad (11)$$

$$\sigma(r) = \frac{e^{-\frac{\lambda}{2}}}{4\pi r} (rE)', \quad (12)$$

where ‘ $'$ ’ denotes differentiation with respect to the radial parameter r . The Eq. (12) yields the expression for electric field as

$$E(r) = \frac{4\pi}{r} \int_0^r r \sigma(r) e^{\frac{\lambda(r)}{2}} dr = \frac{q(r)}{r}, \quad (13)$$

where $q(r)$ is total charge of the sphere under consideration. The generalized Tolman-Oppenheimer-Volkov (TOV) equation for a charged fluid distribution can be written as

$$\frac{1}{2} (\rho + p) \gamma' + p' = \frac{1}{8\pi r^2} (r^2 E^2)', \quad (14)$$

This is the conservation equation in $(2+1)$ -dimensions.

We consider the volume charge density $\sigma(r) e^{\frac{\lambda(r)}{2}}$ (the term inside the integral sign in the Eq. (13)) in the polynomial function of r . Therefore, we can write it in the following form

$$\sigma(r) e^{\frac{\lambda(r)}{2}} = \sigma_0 r^n, \quad (15)$$

where n is arbitrary constant as polynomial index and the constant σ_0 is referred to the central charge density.

By using Eq. (15) in Eq. (13) one can get the solution for $E(r)$ as follows

$$E(r) = \frac{4\pi\sigma_0}{n+2} r^{n+1}, \quad (16)$$

and consequently, one can write

$$q(r) = \frac{4\pi\sigma_0}{n+2} r^{n+2}. \quad (17)$$

Now, the Eq. (9) implies

$$e^{-\lambda(r)} = M(r), \quad (18)$$

where

$$M(r) = C - 16\pi \int r\rho(r)dr - 2 \int rE^2(r)dr - 2 \int r\Lambda dr, \quad (19)$$

is known as the effective gravitational mass of the spherical distribution which determines the gravitational field outside the sphere. Here C is an integration constant.

III. INTERIOR REGION

Seeking interior solution we assume the inheritance symmetry of the spacetime under conformal killing vectors (CKV). CKVs are motions along which the metric tensor of a space-time remains invariant up to a scale factor.

In a given manifold M , we can define a global smooth vector field ξ , known as conformal vector field, such that for the metric g_{ab} it will take the following form

$$\xi_{a;b} = \psi g_{ab} + F_{ab}, \quad (20)$$

where $\psi : M \rightarrow R$ is the smooth conformal function of ξ and F_{ab} is the conformal bivector of ξ . This is equivalent to

$$L_\xi g_{ik} = \psi g_{ik}, \quad (21)$$

where L signifies the Lie derivatives along the CKV ξ^a . Actually CKV provide a deeper insight into the space-time geometry and facilitate the generation of exact solutions to the Einstein's field equations in more comprehensive forms. The study of this particular symmetry in space-time is physically also very important because it plays a crucial role of discovering conservation laws and to devise space-time classification schemes. Also, it is well known that Einstein's field equations are highly non-linear partial differential equations and by using CKV, one can reduce easily the partial differential equations to ordinary differential equations.

The compactified tensorial Eq. (20) yields the following expressions

$$\begin{aligned} \xi^1 \gamma' &= \psi, \\ \xi^0 &= \text{constant}, \\ \xi^1 &= \frac{\psi r}{2}, \\ \xi^1 \lambda' + 2(\xi^1)' &= \psi, \end{aligned}$$

which imply

$$e^\gamma = C_0^2 r^2, \quad (22)$$

$$e^\lambda = \left[\frac{C_1}{\psi} \right]^2, \quad (23)$$

$$\psi = \frac{2\xi^1}{r}, \quad (24)$$

$$\xi^i = C_2 \delta_0^i + \left[\frac{\psi r}{2} \right] \delta_1^i, \quad (25)$$

where the non-zero components of the conformal killing vector ξ^a are ξ^0 and ξ^1 . Here C_0 , C_1 and C_2 all are integration constants.

Now we will study three different cases with different equations of state.

A. $p = m\rho$

By using the equation of state

$$p = m\rho, \quad (26)$$

where $0 < m < 1$ is an equation of state parameter.

From the TOV Eq. (14), we obtain the expressions for the fluid density and pressure as follows:

$$\rho = \frac{4\pi\sigma_o^2}{(n+2)(3m+1+2mn)} r^{2(n+1)} + \frac{A}{m} r^{-\frac{1+m}{m}}, \quad (27)$$

$$p = \frac{4\pi\sigma_o^2 m}{(n+2)(3m+1+2mn)} r^{2(n+1)} + A r^{-\frac{1+m}{m}}, \quad (28)$$

where A is an integration constant.

At this stage, by using Eqs. (16) and (27) in Eq. (19), we get the expression for the effective gravitational mass as

$$M(r) = C - D r^{2(n+2)} + \frac{16\pi A}{m-1} r^{\frac{m-1}{m}} - \Lambda r^2. \quad (29)$$

where the constant D takes the form

$$D = \frac{16\pi^2 \sigma_o^2 (2n+5+3m+2mn)}{(n+2)^3 (3m+1+2mn)}. \quad (30)$$

Then from Eq. (18), after substituting the expression for $M(r)$ from Eq. (29), we get

$$e^{-\lambda} = C - D r^{2(n+2)} + \frac{16\pi A}{m-1} r^{\frac{m-1}{m}} - \Lambda r^2. \quad (31)$$

Also, from Eqs. (23) and (31), we get the following expression for the conformal factor ψ as

$$\psi = C_1 \left(C - Dr^{2(n+2)} - \frac{16\pi A}{m-1} r^{\frac{m-1}{m}} - \Lambda r^2 \right)^{\frac{1}{2}}. \quad (32)$$

One can note that the fluid pressure and density fail to be regular at the origin. However, our demand is that the effective gravitational mass $M(r)$ should always exist as $r \rightarrow 0$. This automatically implies that the integration constant A should be zero. Also, for a physically acceptable model, the energy density should be a decreasing function of radial coordinate r . Therefore Eqs. (27) and (28) indicate the following restriction of n :

$$-2 < n < -1. \quad (33)$$

To be a realistic star model there should have some definite value of the density at the surface. However, effective radial pressure should vanish at the surface $r = R$. This will be taken care of later on.

B. $p = -\rho$

Due to Mazur-Mottola model [4, 5], the spherical fluid contains an isotropic de Sitter vacuum in the interior whereas the exterior is a vacuum black hole solution (in this case, Bañados-Teitelboim-Zanelli (BTZ) type isotropic static charged black hole). In the interior region, let us consider the equation of state

$$p = -\rho. \quad (34)$$

Using this, we obtain the expressions for the fluid density and pressure from the TOV Eq. (14) as follows:

$$p = -\rho = \frac{2\pi\sigma_0^2}{(n+2)(n+1)} r^{2(n+1)} + H, \quad (35)$$

where H is an integration constant.

Here, we get the following expression for $M(r)$ as well as $e^{-\lambda}$ as

$$e^{-\lambda} = M(r) = C + Lr^{2(n+2)} - 8\pi Hr^2 - \Lambda r^2 \quad (36)$$

where,

$$L = \frac{16\pi^2\sigma_0^2}{(n+1)(n+2)^3} \quad (37)$$

As before, since ρ is decreasing with increasing r and $M(r)$ is regular at the origin, one should impose the restriction on n as before $-2 < n < -1$.

Keeping in mind the notion of gravastar, we consider apart from the interior region, that the configuration has a region of thin shell. One may consider that this thin shell region (Fig. 1) contains matter having equation of state

$$p = \rho. \quad (38)$$

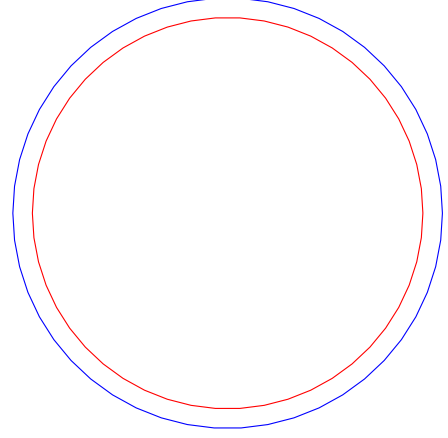


FIG. 1: The shell with thickness ϵ contains matter satisfying the equation of state $p = \rho$

Here, the radius of the interior region is assumed to be R and thickness of the shell is ϵ , no matter how small it is. This means that outer radius of the fluid configuration is $R + \epsilon$.

Using the equation of state (38), we get the following solutions from the TOV Eq. (14) and Eq. (19) as

$$p = \rho = \frac{2\pi\sigma_0^2}{(n+2)^2} r^{2(n+1)} + Nr^{-2}, \quad (39)$$

where N is an integration constant.

The metric potential $e^{-\lambda}$ can be written as

$$e^{-\lambda} = M(r) = C - Ur^{2(n+2)} - 16\pi N \ln r - \Lambda r^2, \quad (40)$$

where

$$U = \frac{32\pi^2\sigma_0^2}{(n+2)^3}. \quad (41)$$

Imposing the decreasing criteria of density, we take $n < -1$. Here range of r is $R \leq r \leq R + \epsilon$. Therefore, all the physical parameters are regular everywhere within the shell.

IV. EXTERIOR REGION OF CHARGED BTZ-TYPE BLACK HOLE

Here we have considered that the electro-vacuum exterior solution ($p = \rho = 0$) corresponds to a static, charged BTZ-type black hole is written in the following form [2]

$$ds^2 = -(-M_0 - \Lambda r^2 - Q^2 \ln r) dt^2 + (-M_0 - \Lambda r^2 - Q^2 \ln r)^{-1} dr^2 + r^2 d\theta^2, \quad (42)$$

where Q is the total charge and M_0 is the conserved mass of the black hole which is associated with asymptotic invariance under time-displacements. This mass is given by a flux integral through a large circle at space-like infinity.

V. MATCHING CONDITION

Let us assume R be the radius of the charged fluid sphere. Then continuity of the metric functions g_{tt} and g_{rr} across the surface of charged fluid sphere yields the following results:

$$C_0^2 R^2 = -M_0 - Q^2 \ln R - \Lambda R^2, \quad (43)$$

$$C - DR^{2(n+2)} - \Lambda R^2 = -M_0 - Q^2 \ln R - \Lambda R^2. \quad (44)$$

We get the values for integrating constants C_0 and C as

$$C_0 = \frac{1}{R}(-M_0 - \Lambda r^2 - Q^2 \ln R)^{\frac{1}{2}}, \quad (45)$$

$$C = DR^{2(n+2)} - M_0 - Q^2 \ln R. \quad (46)$$

As the effective radial pressure ($p_{eff} = \frac{\gamma' e^{-\lambda}}{2r}$) should vanish at the boundary of the charged fluid sphere, therefore from Eq. (10), we get

$$\Lambda = -\frac{16(1-m)\pi^2\sigma_0^2 R^{2n+2}}{(n+2)^2(3m+1+2mn)}. \quad (47)$$

Since, m lies between 0 and 1, so Eq. (47) at once confirms that the cosmological constant should be negative. This equation, in turn, yields the radius of the star as

$$R = \left[\frac{-\Lambda(n+2)^2(3m+1+2mn)}{16(1-m)\pi^2\sigma_0^2} \right]^{\frac{1}{2n+2}}. \quad (48)$$

From Eq. (30), we get (considering $P_{eff} = \frac{\gamma' e^{-\lambda}}{2r} = 0$, at the boundary $r = R$) another expression for Λ given by

$$\Lambda = \frac{C}{R^2} - DR^{2(n+1)}. \quad (49)$$

Definitely, Eqs. (47) and (49) should be equal. Then, equating these two equations and after substituting the value of D from Eq. (30), one obtains an expression for C as follows:

$$C = 16\pi^2\sigma_0^2 \left[\frac{(n+3+5m+3mn)}{(n+2)^3(3m+1+2mn)} \right] R^{2n+4}, \quad (50)$$

which is another form of the equation (46).

Therefore, after substituting C and D from Eqs. (50) and (30), we get the total effective gravitational mass from Eq. (29) in the form

$$M(R) = \frac{16\pi^2\sigma_0^2(m-1)}{(n+2)^2(3m+1+2mn)} R^{2n+4} - \Lambda R^2. \quad (51)$$

One can note from the above Eq. (51) that in the absence of Λ (see Eq. (47)), the total effective gravitational mass $M(R)$ becomes negative as constraint on the equation of state parameter is $0 < m < 1$. This means inclusion of Λ is unavoidable for the present model to have a usual positive gravitational mass.

Again, from the Eqs. (46) and (50) we can find out the expression for the central charge density, σ_0 , in terms of Q , R and M_0 as given by

$$\sigma_0^2 = (M_0 + Q^2 \ln R) \left[\frac{(n+2)^2(3m+1+2mn)}{16(1-m)\pi^2} \right] R^{-(2n+4)}. \quad (52)$$

Thus it is evident from the above Eq. (52) that the central charge density is dependent on the value of M_0 , the conserved mass of the BTZ black hole. This in turn depends on the black hole event horizon which again is related to the Hawking radiation temperature of a BTZ black hole. Thus one may have a clue that the central charge density is related with Hawking radiation temperature of the BTZ black hole on the exterior region of the static charged fluid sphere.

VI. PHYSICAL ANALYSIS

We provide here first order derivatives of ρ to show variational nature of this parameter by assuming from the very beginning the constraint $A = 0$, as explained earlier. This is:

$$\frac{d\rho}{dr} = \frac{8\pi\sigma_0^2(n+1)}{(n+2)(3m+1+2mn)} r^{2n+1} \quad (53)$$

One can note that $\frac{d\rho}{dr}$ is negative since $n < -1$. This again supports that ρ is decreasing in nature. In addition, we would also recall the equation of state, and observe from the Eq. (25) that the sound velocity, $v_s^2 = \frac{d\rho}{dp} = m < 1$, which indicates that the model is realistic.

VII. CONCLUSION

In this paper we have presented a new solution set for a static spherically symmetric fluid distribution under the frame work of Einstein-Maxwell space-time. As an additional condition we have allowed CKV to find out inheritance symmetry of the system of (2+1)-dimensional gravity associated with isotropic fluid. We further consider exterior solution of BTZ-type static, charged black hole so that our interior solution can be matched smoothly as a consequence of junction conditions.

Beside the general physical properties these solutions set shows two special features: (i) *gravastar* like configuration, and (ii) *electromagnetic* origin of gravitational sources.

In connection to non-singularity we notice from the Eq. (29) that the effective gravitational mass exists at $r = 0$, the constant C (Eq. (50)) being a non-zero quantity. On the other hand, all the physical parameters M , p , ρ , Λ etc. completely depend on the charge density σ , such that for vanishing charge (Eq. (17)) the charge density vanishes, which in turn makes all the parameters vanish. This type of solution is known in the literature as *Electromagnetic Mass* model [17–20]. Interestingly our model also represents a charged *Gravastar* as an alternative of black hole as according to Mazur and Mottola's [4, 5] concept the fluid sphere may contain three different regions with different equations of state (Interior: $0 \leq r < r_1$, $p = -\rho$; (II) Shell: $r_1 < r < r_2$, $p = +\rho$; and (III) Exterior: $r_2 < r$, $p = \rho = 0$). The mechanism to achieve this situation due to inclusion of charge is available in the literature [7–11].

From the solution set of the present studies we observe some of the following interesting features which raise deeper issues to consider for further investigations:

(1) In Eq. (47) we have Λ , the cosmological constant to be negative from our calculations. Actually this seems quite plausible since we have considered the exterior region to be that of a BTZ-type black hole which has been solved here with a negative cosmological constant. Here Λ have been assumed to be a constant as was adopted by Einstein for his static cosmological model. But now there are enough evidence in favour of an accelerating Universe

[21, 22], which demands Λ to be a dynamical parameter, a candidate of so called *dark energy*, energy of vacuum. So, a proper follow up scheme of the present work is to consider further a time varying phenomenological Λ term. Then the radius of the charged fluid sphere can be shown to vary accordingly. If we can conclude that the interior solution of the charged fluid sphere is analogous to that of a charged gravastar admitting inheritance symmetry then we can have a perfect correspondence of the radius of the gravastar with dark energy.

(2) We note that our negative cosmological constant Λ (Eq. (47)) can be expressed in terms of the radius of curvature, $l = (-\Lambda)^{-\frac{1}{2}}$ [1, 23]. The Bekenstein-Hawking entropy associated with BTZ type of black hole is actually twice the perimeter of the outer horizon, $S = 4\pi r_+$ [1, 23]. This can be linked with the characteristic Hawking radiation temperature. So, we see a possibility that entropy and hence radiation temperature of the BTZ type charged black hole can be studied which will be done elsewhere in a future project.

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